

Quantitative Risk Management: VaR and Others

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What is Value At Risk?

- Value at Risk, or VaR, is roughly speaking, a measure of how much money a bank or other financial firm can lose on its positions in a fixed period, such as 1 day, 10 days, or 1 year in a “worst case” (bottom 1 percent) scenario.
- Losses can be due to diffusive moves (“general” VaR) or defaults or credit migrations (“incremental risk charge”), or so-called “special events” (“specific risk”)

What is VaR Used For?

- VaR is used as a tool to decide how large a trading book to have. An equities or FX desk will often have an overall VaR limit enforced by upper management. A “real-time” VaR calculation can determine whether a trade is possible.
- VaR is used at the firm level to determine the amount of capital the Feds will require the firm to have. VaR capital is combined with capital requirements from Specific Risk, Stress Scenarios and other risk measures mentioned here.

Basic Calculation Methods

- Historical Simulation – last 251 1 or 10-day interval market changes are applied to current conditions, and we take the second worst profit or (more likely) loss.
- Monte Carlo Model – Create a parametric model for the next period's moves based on current model and last year's historical data, simulate N times, take top of bottom 1% of P&Ls

General vs Stress VaR

- General VaR estimates bad case 1 or 10 day loss based on the previous year's history
- Stress VaR estimates bad case 1 or 10 day loss based on the market moves that occurred during a particularly “stressful” year, such as Sept 2008 to Sept 2009.

Historical General VaR, More Detail

- Bank's positions defined as $P_j, j = 1, \dots, N$.
- Each position has risk factors $\{X_{1,j}, X_{2,j}, \dots, X_{n_j,j}\}$ chosen primarily from market observable inputs, not from underlying calibrated parameters.
- Each risk factor has historical time series $X_{i,j}(t)$ and 1-day shifts $\Delta X_{i,j,t} = X_{i,j}(t) - X_{i,j}(t - 1)$ (absolute) or $\Delta X_{i,j,t} = (X_{i,j}(t) - X_{i,j}(t - 1))X_{i,j}(0)/X_{i,j}(t - 1)$ (relative).
- For each t , each position indexed by j , define $\Delta P_j(t) = P(X_{1,j}(0) + \Delta X_{1,j}(t), \dots, X_{n_j,j}(0) + \Delta X_{n_j,j}(t)) - P(X_{1,j}(0), \dots, X_{n_j,j}(0))$.

Historical General VaR, cont'd

- Total P&L across all positions $\Delta P(t) = \sum_{j=1}^N \Delta P_j(t)$
- Sort all 251 $\Delta P(t)$'s one for each business day over the last year, from low to high, and take the second one in the sorted list.
- Not really the first percentile, really closer to the 0.8 percentile, but the Feds do not allow interpolating between the 2nd and 3rd worst P&L.

One Day versus Ten Days

- 10-day shifts $\Delta X_{i,j,t} = X_{i,j}(t) - X_{i,j}(t - 10)$
(absolute) or
 $\Delta X_{i,j,t} = (X_{i,j}(t) - X_{i,j}(t - 10))X_{i,j}(0) / X_{i,j}(t - 10)$ (relative).
- Shifts overlap and cover 260 business days total.

Delta/Gamma Approximation

- Historical simulation with full revaluation requires 252 pricings for each position, current price plus effect of 251 different shifts.
- We may save time by using the estimate

$$\Delta P_j(t) \approx \sum_{i=1}^{n_j} \left(\frac{\partial P_j}{\partial X_{i,j}} \Delta X_{i,j,t} + \frac{1}{2} \frac{\partial^2 P_j}{\partial X_{i,j}^2} \Delta X_{i,j,t}^2 \right) + \sum_{i < k} \frac{\partial^2 P_j}{\partial X_{i,j} \partial X_{k,j}} \Delta X_{i,j,t} \Delta X_{k,j,t}$$

It is common to leave out the cross terms since they are often very small. Cross terms are a form of Risk Not in VaR (RNiV), to be covered in a later slide.

Delta/Gamma Cont'd

Greeks estimated as

$$\frac{\partial P_j}{\partial X_{i,j}} = \frac{P_j(X_{1,j}, \dots, X_{i,j} + h, \dots, X_{n_j,j}) - P_j(X_{1,j}, \dots, X_{i,j} - h, \dots, X_{n_j,j})}{2h}$$

$$\frac{\partial^2 P_j}{\partial X_{i,j}^2} = \frac{P_j(X_{1,j}, \dots, X_{i,j} + h, \dots, X_{n_j,j}) - 2P_j + P_j(X_{1,j}, \dots, X_{i,j} - h, \dots, X_{n_j,j})}{h^2}$$

For relative bump sizes replace h with $hX_{i,j}$.

Most Accurate Bump Sizes

- Absolute: $h_{i,j} \approx s.d.(\Delta X_{i,j})$
- Relative: $h_{i,j} \approx \text{one period vol}(X_{i,j})$

Calibrated Parameters vs Risk Drivers

- The risk drivers for VaR, the X 's, are typically market inputs, e.g. spot, ATM volatility, some sort of volatility skew measure, and interest rates, rather than underlying model parameters.
- Full revaluation would require re-calibration for every historical shift of the inputs.
- Delta Gamma approximation requires inverting a Jacobian matrix (example next slide)

Calibrated Parameters vs Risk Drivers

Let $(X_{1,j}, \dots, X_{n_j,j}) = G(Y_{1,j}, \dots, Y_{n_j,j})$. Then

$$\left(\frac{\partial P_j}{\partial X_{1,j}}, \dots, \frac{\partial P_j}{\partial X_{n_j,j}} \right) = J^{-1} \left(\frac{\partial P_j}{\partial Y_{1,j}}, \dots, \frac{\partial P_j}{\partial Y_{n_j,j}} \right)^T$$

Example would be if we priced an option using a Heston model with spot, instantaneous vol, correlation and vol of vol, but risk drivers are spot, ATM vol, risk reversal and butterfly. Second derivatives are messier but can be worked out.

Grid Approximation

- Usually done on underlying price risk factor only, so consider the case of just this risk factor.
- Compute $P_j(X - h_{-n}), \dots, P_j(X), \dots, P_j(X - h_n)$.
- Suppose that $\Delta X_{j,t} = (1 - \rho)h_k + \rho h_{k+1}$
- $\Delta P_j = (1 - \rho)P_j(X + h_k) + \rho P_j(X + h_{k+1}) - P_j(X)$
- For additional drivers, add the delta/gamma terms.

Expected Shortfall

- VaR takes the second worst of 251 P&Ls for the total position.
- Expected shortfall computes the average of the 6 or 7 worst P&Ls for the total position.
- Some banks use this in lieu of or in addition to VaR because it appears to be more stable.
- Regulators will eventually require it.

Common risk drivers

- Equities – stock price, discount rate, ATM volatilities by tenor, volatility skew and convexity.
- FX – spot, ATM volatility, foreign and domestic rates, risk-reversals and butterflies
- Commodities – spot, ATM volatility, various measures of skew
- Interest Rates – forward swap rates, ATM volatilities, volatility skew for each expiry and tenor.

Monte Carlo Method

- In this case we postulate a stochastic process for the inputs to the price of the position.
- May be similar in structure to front office pricing model, except that model must reflect past 1-year history rather than strictly calibrate to market data.
- Instead of 251 P&Ls for each position, we can simulate 5000, or 50000 or more scenarios.
- VaR is set to 1st percentile, i.e. 500th worst out of 50000.

Stress VaR

- Similar to General VaR, except that historical shifts in risk factors are taken in a particular “bad” 1-year period in the past, such as Sept 2008 to Sept 2009, instead of the most recent 1-year period.
- Stress period does not change frequently.
- t ranges over business days $T, T + 1, \dots, T + 251$, for some T at the beginning of the stress period, rather than $-252, -251, \dots, -1$ (yesterday's close).

Stress Scenarios

- Reflects a particularly bad interval in the past, just two times (t_1, t_2) .
- Examples include 9/11, Lehman bankruptcy.
- Usually computed separately for several different scenarios.
- Compute $\Delta P_j = P_j \left(X'_{1,j}, \dots, X'_{n_j,j} \right) - P_j \left(X_{1,j}, \dots, X_{n_j,j} \right)$
- $X'_{i,j} = X_{i,j} + X_{i,j}(t_2) - X_{i,j}(t_1)$ absolute
- $X'_{i,j} = X_{i,j} * X_{i,j}(t_2) / X_{i,j}(t_1)$ relative

Risk not in VaR (RNiV)

- Models for exotics usually depend on trader-marked parameters which are estimated and don't change frequently
- Examples include correlation or correlation skew for basket options, mean-reversion in commodity markets, and correlation between spot and volatility
- Cross terms in delta-gamma approximation

RNiV, continued

- Situations where shifting one driver causes another to move (extra terms in the chain rule, double-counting).
- Mismatch between driver sensitivity and actual historical time series.
- The regulators require us to keep track of sources of Risk not in VaR.

Specific Risk (SR)

- Specific risk “picks up where VaR leaves off”, meaning that it attempts to capture rare events not likely in a 1-year period.
- Examples include huge moves in bond price, stock price or volatility caused by credit migrations, scandals, wars, drug non-approvals, etc.
- Specific risk is made up of Debt Specific Risk (DSR) and Equity Specific Risk (ESR)

Debt Specific Risk

- VaR for sovereign, agency, municipal or corporate bonds and credit default swaps captures the risk from moves in the generic credit spread for that instrument's rating.
- DSR attempts to capture risk from the “idiosyncratic” part of the credit spread, i.e. the difference between the credit spread for that CDS (or bond) and the contribution from the generic spread from that instrument's rating.

DSR, Continued

- Run regression on each bond between overall spread and generic spread:

$$\Delta s_i = \beta_i \Delta g_i + \Delta s_i^{idio}$$

$$E(\Delta s_i^{idio}) = \mu_i$$

$$s.d.(\Delta s_i^{idio}) = \sigma_i$$

The index i is for the bond or CDS.

DSR, continued

Then we average over all the bonds and over all the tenors to get a average volatility and mean for a single idiosyncratic spread. Then we simulate it by Monte Carlo and compute 500,000 P&Ls of the form $\left(\frac{\partial P}{\partial s}\right) \Delta s^{idio}$. Then we sort the total P&Ls and take number 5000.

Equity Specific Risk (ESR)

- ESR attempts to capture events that may occur over a 10-year period, rather than a 1-year period.
- For each name, compute the 1-year historical vol and 10 year kurtosis.
- Create a random variable with that volatility and kurtosis
- Use principal component analysis to correlate all the different names' random variables.
- Simulate 500,000 times for each name, and zero P&L contributions out any returns less than 2.6 standard deviations
- Add all the P&Ls over all the positions, sort and take Number 5000 as the specific risk.

ESR Model – A Simple Example

- There are N stock indices, and each index has a return of the form

$$r_i = \sqrt{\xi} (a_{i,1}Z_1 + a_{i,2}Z_2 + \cdots + a_{i,M}Z_M)$$

$$Z_j \sim N(0,1), \text{ all independent}$$

$$E(\xi) = 1, E(\xi^2) > 1, \text{ independent from } Z\text{'s}$$

$$Kurt(r_i) = 3(E(\xi^2) - 1)$$

The Z 's represent the principal components of the market.

Individual Stock Treatment

$$r_s = br_I + \sigma_s \sqrt{\xi_s} Z_s$$

ξ, Z, r_I all independent

$$\text{Let } \sigma_I^2 = \text{var}(r_I)$$

$$\text{Then } \text{var}(r_s) = b\sigma_I^2 + \sigma_s^2$$

$Kurt(r_s)$

$$= 3 \frac{b^4 \sigma_I^4 (E(\xi_I^2) - 1) + \sigma_s^4 (E(\xi_s^2) - 1)}{b^4 \sigma_I^4 + 2b^2 \sigma_I^2 \sigma_s^2 + \sigma_s^4}$$

Changes in Volatility

We also compute the average implied volatility change, and model it as

$$\Delta(iv) = v \left(\rho \frac{r}{\sigma_r} + \sqrt{1 - \rho^2} \sqrt{\xi_{iv}} Z_{iv} \right)$$

where ρ is the correlation between implied volatility and equity index return. Once again, we choose random variable ξ_{iv} to have

$$E(\xi_{iv}) = 1, E(\xi_{iv}^2) > 1.$$

P&L Contributions

- We only keep “large enough” P&L contributions from either price or volatility
- Price: $\Delta P = P(X(1 + r_k), \dots) - P(X, \dots)$ if $|r_k| > 2.6 * s.d. (r)$, 0 otherwise. k is the path index. Calculated by grid approximation.
- Implied Volatility: $\Delta P = P(\dots, IV + \Delta(IV), \dots) - P(\dots, IV, \dots)$ if $\Delta(IV) > 2.6 * s.d. (\Delta(IV))$, 0 otherwise.
- There are both 1-day and 10-day versions of ESR.

ESR Shortcomings

- Difficult to determine the best threshold for defining “event” risk (we use 2.6 standard deviations, but this is hard to calculate).
- Returns and changes in implied volatility follow a distribution which has kurtosis, but not skew. Difficult to allow for skew in this general framework.

Incremental Risk Charge (IRC)

- Aims to capture impact on the credit book (bonds and CDSs) from migrations and defaults, which are not included in VaR or DSR.
- Create up to 3 $N(0,1)$ random variables based on market, regional and sector returns.
- Use these to create $N(0,1)$ random variables for each name, with the desired correlation matrix.
- Default or migration for each name can occur in 1 year if the variable breaches any of a set of thresholds.

IRC, continued

- Recovery is a function of each name's random variable
- Each simulation is a single 1-year drawing for the underlying variables
- For each position, compute a P&L only if a migration or default occurs (most are 0).
- Add up total P&L, sort, and choose number 500 out of 500,000 simulations (99.9% confidence level, rather than 99%)

Some recent and on going regulatory requirements and modeling projects

- Incorporate changes in volatility skew and convexity as a function of strike as risk factors in Equity, Interest Rate, FX and Commodity VaR.
- Improve internal pricing models used for VaR for bonds.
- Eventually move toward full revaluation rather than delta/gamma and grid approximations.

Ongoing projects, cont'd

- Incorporate dividend risk in Equity VaR.
- Compute CVA in a manner consistent with front office models and then compute CVA VaR (this is extremely difficult).
- Compute risk for private equity positions
- Estimate impact of Risk Not in VaR by varying marked parameters.